

CONTACT MANIFOLDS ALL OF WHOSE REEB ORBITS ARE CLOSED

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Reeb flows are a remarkable class of dynamical systems on contact manifolds. Examples of such flows are the geodesic flows on unit tangent bundles of Riemannian manifolds, and more generally certain Hamiltonian flows on energy hypersurfaces of symplectic manifolds. In Riemannian geometry, Besse geodesic flows have been thoroughly studied since the work of Zoll, Bott, Samelson, Berger, and many other authors. In contact geometry, Besse Reeb flows were first considered by Boothby, Wang, and Thomas.

In this series of lectures, after introducing the relevant notions, I will focus on those Reeb flows all of whose orbits are closed, which are usually called Besse Reeb flows. I will then address the following conjecture: *One can recognize whether a given Reeb flow is Besse from its action spectrum, that is, from the set of periods of its closed Reeb orbits.* This conjecture is currently confirmed for Riemannian 2-spheres and general closed 3-manifolds. For Reeb flows on certain higher dimensional contact manifolds, weaker statements are known. The goal of the minicourse will be to give an overview of the state of the art and of the techniques involved in the proofs.

The lectures will be mostly self-contained. The interested students will be assumed to have some background in differential geometry and dynamical systems (manifolds, vector fields, flows, and some basic algebraic topology), but not on symplectic and contact geometry.

Tentative program

- Lecture 1: Lusternik-Schnirelmann theory, geodesic flows, closed geodesics, Besse and Zoll geodesic flows.
- Lecture 2: Spectral characterization of Zoll geodesic flows on the 2-sphere and, time permitting, on higher dimensional manifolds.
- Lecture 3: Besse and Zoll contact forms, spectral characterization of Besse contact convex spheres
- Lecture 4: Spectral characterization of Besse contact 3-manifolds.

References

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- D. Cristofaro-Gardiner and M. Mazzucchelli, *The action spectrum characterizes closed contact 3-manifolds all of whose Reeb orbits are closed*, arXiv:1901.10617, 2019.
- H. Geiges, *An introduction to contact topology*, Cambridge Studies in Advanced Mathematics, vol. 109, Cambridge University Press, Cambridge, 2008.
- M. Mazzucchelli and S. Suhr, *A characterization of Zoll Riemannian metrics on the 2-sphere*, Bull. Lond. Math. Soc. **50** (2018), 997–1006.
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